

## Pauli-limited upper critical field in dirty $d$ -wave superconductors

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We calculate the Pauli-limited upper critical field and the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) instability for *dirty*  $d$ -wave superconductors within the quasiclassical theory using the self-consistent  $\hat{t}$ -matrix approximation for impurities. We find that the phase diagram depends sensitively on the scattering rate and phase shift of nonmagnetic impurities. The transition into the superconducting state is always second order for weak (Born) scattering, while in the unitarity (strong) scattering limit a first-order transition into both uniform and spatially modulated superconducting states is stabilized. Contrary to general belief, we find that the FFLO phase is robust against disorder and survives impurity scattering equivalent to a  $T_c$  suppression of roughly 40%. Our results bear on the search of FFLO states in heavy-fermion and layered organic superconductors.

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In type-II singlet superconductors a magnetic field suppresses superconductivity for two reasons: (1) the phase of the Cooper pair wave function couples to the vector potential resulting in the appearance of vortices; and (2) Zeeman coupling of the magnetic field to the electron spins polarizes and splits the conduction band, which destroys superconductivity when the loss in magnetic energy equals the energy gain from pair condensation.<sup>1-4</sup> This latter mechanism is referred to as Pauli limiting and leads to a first- or second-order transition from the normal (N) to superconducting (SC) state depending on the value of the magnetic field. It has been predicted that a clean system at high fields can remain superconducting beyond the Pauli limit by forming the nonuniform Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state with a spatially modulated order parameter.<sup>5</sup> This state, however, is suppressed by disorder.<sup>6</sup>

In contrast to conventional (isotropic  $s$ -wave) superconductors, unconventional ( $d$ -wave) superconductors are affected by nonmagnetic impurities even at zero field; scattering averages the gap over the Fermi surface and suppresses  $T_c$ . The different rates of suppression of the uniform and FFLO states determine the phase diagram in the field-temperature ( $B$ - $T$ ) plane. Agterberg and Yang<sup>7</sup> found that in two-dimensional (2D)  $d$ -wave superconductors with purely Zeeman coupling, the N-SC transition is of second order at all  $T$ , with the Larkin-Ovchinnikov (LO) modulation,  $\Delta_{LO} \sim \cos \mathbf{q} \cdot \mathbf{R}$ , and the uniform (USC) state,  $\Delta_{USC} = \text{const}$ , favored at low and high temperatures, respectively, and a narrow intermediate  $T$  region, where the nodeless Fulde-Ferrell (FF) state,  $\Delta_{FF} \sim e^{i\mathbf{q} \cdot \mathbf{R}}$ , is stabilized. Reference 8 reported that under combined orbital and Zeeman coupling in impure  $d$ -wave superconductors the first-order transition into the vortex state appears at intermediate temperatures. Very recently, Houzet and Mineev<sup>9</sup> studied orbital and impurity effects in  $s$ -wave and  $d$ -wave Pauli-limited superconductors and concluded that orbital effects are necessary for a first-order transition to occur in 2D  $d$ -wave superconductors. In contrast, for  $s$  wave in 3D the transition to the FFLO state is first order.<sup>10,11</sup>

So far our understanding of impurity effects in nonuniform states is still incomplete. References 7-9 analyzed weak (Born) impurity scattering and focused only on the Ginzburg-Landau (GL) regime close to the onset of the

FFLO instability, using an expansion in the modulation wave vector  $\mathbf{q}$ . However,  $q = |\mathbf{q}|$  increases rapidly to values comparable to the inverse superconducting coherence length,  $q\xi_0 \sim 1$ , so this expansion quickly becomes invalid away from the critical point.

In this Rapid Communication, we present a microscopic treatment of impurity effects in Pauli-limited 2D  $d$ -wave superconductors. We treat this case since (a) the structure of the FFLO-vortex state at low  $T$  is not known and (b) impurities destroy FFLO modulation but weakly affect orbital physics. Impurities are treated in the self-consistent  $\hat{t}$ -matrix approximation (SCTA) covering the weak (Born) and strong (unitarity) scattering limits.<sup>12</sup> The latter limit, never considered previously, is especially important because of a search for FFLO-like states in heavy-fermion and layered organic superconductors,<sup>13</sup> where impurity scattering phase shifts are large.<sup>14</sup> Our approach is not limited to an expansion in  $q$ , and hence is valid for any temperature and impurity concentration along the second-order upper critical field  $B_{c2}$ . We show that the phase diagram of a Pauli-limited *dirty*  $d$ -wave superconductor is very different for nonmagnetic impurities in the Born and unitarity limits. The differences originate from the dependence on scattering strength of quartic and higher order coefficients in the GL functional. The first-order N-SC transition, absent for Born scattering, is stabilized by strong impurities, and is therefore expected in heavy fermion systems.

We follow Refs. 12 and 15 and solve the quasiclassical equations for the  $4 \times 4$ -matrix Green's functions in particle-hole and spin space, which satisfy the normalization condition,  $\hat{g}^2 = -\pi^2 \hat{1}$ , and the transport equation,

$$[i\varepsilon_m \hat{\tau}_3 - \mu \mathbf{B} \cdot \hat{\mathbf{S}} - \hat{\Delta}(\mathbf{R}, \hat{\mathbf{p}}) - \hat{\sigma}^{\text{imp}}(\mathbf{R}; \varepsilon_m), \hat{g}(\mathbf{R}, \hat{\mathbf{p}}; \varepsilon_m)] + i\hbar \mathbf{v}_f(\hat{\mathbf{p}}) \cdot \nabla_{\mathbf{R}} \hat{g}(\mathbf{R}, \hat{\mathbf{p}}; \varepsilon_m) = 0. \quad (1)$$

Here  $\mu$  is the magnetic moment,  $\varepsilon_m = \pi k_B T(2n+1)$  are the Matsubara frequencies, and  $\hat{\Delta}$  is the mean-field superconducting order parameter depending on the coordinate  $\mathbf{R}$  and momentum direction  $\hat{\mathbf{p}}$  at the Fermi surface with velocity  $\mathbf{v}_f$ . The electron spin operator is  $\hat{\mathbf{S}} = \boldsymbol{\sigma} \frac{1}{2} (1 + \hat{\tau}_3) + \boldsymbol{\sigma}^* \frac{1}{2} (1 - \hat{\tau}_3)$ . The Pauli matrices  $\boldsymbol{\sigma}$  and  $\boldsymbol{\tau}$  operate in spin and particle-hole space, respectively. Equation (1) is complemented by self-

consistency equations for  $\hat{\Delta}$  and the impurity self-energy  $\hat{\sigma}^{\text{imp}}$ . We use  $\hbar = k_B = 1$ .

In the SCTA  $\hat{\sigma}^{\text{imp}} = n_{\text{imp}} \hat{t}$ , with impurity concentration  $n_{\text{imp}}$ . For isotropic scattering the  $t$  matrix satisfies  $\hat{t}(\mathbf{R}; \varepsilon_m) = u_0 \hat{1} + u_0 \mathcal{N}_f \langle \hat{g}(\mathbf{R}, \hat{\mathbf{p}}; \varepsilon) \rangle_{\hat{\mathbf{p}}} \hat{t}(\mathbf{R}; \varepsilon_m)$ , where angular brackets  $\langle \cdots \rangle$  denote a normalized Fermi surface average. The strength of the nonmagnetic impurity potential,  $u_0$ , is expressed via the isotropic scattering phase shift,  $\delta_0 = \arctan(\pi u_0 \mathcal{N}_f)$ ;  $\mathcal{N}_f$  is the density of states per spin at the Fermi surface. For Born (unitarity) scattering  $\delta_0 = 0$  ( $\delta_0 = \pi/2$ ) and the normal-state scattering rate  $\Gamma \equiv 1/2\tau_N = \Gamma_u \sin^2 \delta_0$ , with  $\Gamma_u = n_{\text{imp}} / \pi \mathcal{N}_f$ .

If we choose the direction of the spin quantization along  $\mathbf{B} = B\hat{\mathbf{z}}$  (which is allowed if the Hamiltonian has spin-rotation symmetry in the absence of the field), both  $\hat{g}$  and  $\hat{\sigma}^{\text{imp}}$  have block-diagonal structure corresponding to the two spin projections. Hence, the quasiclassical equations for the spin-up and spin-down sectors decouple,<sup>16</sup> and we solve separately for the diagonal,  $g_s$ , and off-diagonal,  $f_s, f'_s$  components of  $\hat{g}$ , with  $s = \pm 1\{\uparrow, \downarrow\}$ , with the constraint  $g_s^2 - f_s f'_s = -\pi^2$ . However, both spin projections enter the self-consistency equa-

tion for  $\hat{\Delta}$ . We assume a separable pairing interaction  $\mathcal{Y}(\hat{\mathbf{p}})\mathcal{Y}(\hat{\mathbf{p}}')$ , where  $\mathcal{Y}(\hat{\mathbf{p}})$  gives the angular dependence of the gap function with the normalization  $\langle \mathcal{Y}^2(\hat{\mathbf{p}}) \rangle = 1$ . For  $\Delta(\mathbf{R}, \hat{\mathbf{p}}) = \Delta(\mathbf{R})\mathcal{Y}(\hat{\mathbf{p}})$ , we find

$$\Delta(\mathbf{R}) \ln \frac{T}{T_{c0}} = T \sum_{\varepsilon_m} \left( \langle \mathcal{F}(\mathbf{R}, \hat{\mathbf{p}}; \varepsilon_m) \rangle_{\hat{\mathbf{p}}} - \frac{\pi \Delta(\mathbf{R})}{|\varepsilon_m|} \right), \quad (2)$$

$$\hat{\sigma}_s^{\text{imp}} = \mathcal{S}_s \begin{pmatrix} \cot \delta_0 + \langle g_s \rangle / \pi & \langle f_s \rangle / \pi \\ \langle f'_s \rangle / \pi & \cot \delta_0 - \langle g_s \rangle / \pi \end{pmatrix}. \quad (3)$$

Here  $\mathcal{F}(\mathbf{R}, \hat{\mathbf{p}}; \varepsilon_m) = \frac{1}{2} \mathcal{Y}(\hat{\mathbf{p}}) [f_{\uparrow}(\mathbf{R}, \hat{\mathbf{p}}; \varepsilon_m) + f_{\downarrow}(\mathbf{R}, \hat{\mathbf{p}}; \varepsilon_m)]$  and  $\mathcal{S}_s = \Gamma / [1 - \pi^{-2} \sin^2 \delta_0 (\langle g_s \rangle^2 - \langle f_s \rangle \langle f'_s \rangle + \pi^2)]$ . To calculate the  $B$ - $T$  phase diagram, we derive the Ginzburg-Landau functional (expansion in  $\Delta$  for arbitrary  $q$ ) by taking  $\Delta(\mathbf{R}) = \sum_{\mathbf{q}} \Delta_{\mathbf{q}} \exp(i\mathbf{q} \cdot \mathbf{R})$  and solving Eqs. (1)–(3) together with the normalization condition for  $\hat{g}$  to third order in  $\Delta$ . We substitute the  $n$ th order solutions  $f_s^{(1)}, f_s^{(3)}$  into Eq. (2) to obtain the GL free energy difference between the SC and N states,

$$\Delta \Omega^{\text{GL}} = \sum_{\mathbf{q}} \alpha(T, B; \mathbf{q}) |\Delta_{\mathbf{q}}|^2 + \sum_{\mathbf{q}_1 \mathbf{q}_2 \mathbf{q}_3 \mathbf{q}_4} \frac{1}{2} \beta(T, B; \mathbf{q}_1, \mathbf{q}_2; \mathbf{q}_3, \mathbf{q}_4) \Delta_{\mathbf{q}_1} \Delta_{\mathbf{q}_2} \Delta_{\mathbf{q}_3}^* \Delta_{\mathbf{q}_4}^* \delta_{\mathbf{q}_1 + \mathbf{q}_2, \mathbf{q}_3 + \mathbf{q}_4}, \quad (4a)$$

$$\alpha(T, B; \mathbf{q}) = \ln \frac{T}{T_{c0}} - 2\pi T \sum_{\varepsilon_m > 0} \text{Re} \left( \langle \mathcal{Y} \tilde{\mathcal{Y}}_{\mathbf{q}} D_{\mathbf{q}}^{-1} \rangle - \varepsilon_m^{-1} \right), \quad (4b)$$

$$\beta(T, B; \mathbf{q}_1, \mathbf{q}_2; \mathbf{q}_3, \mathbf{q}_4) = \pi T \sum_{\varepsilon_m > 0} \text{Re} \left\{ \left\langle \tilde{\mathcal{Y}}_{\mathbf{q}_1} \tilde{\mathcal{Y}}_{\mathbf{q}_2} \tilde{\mathcal{Y}}_{\mathbf{q}_3} \tilde{\mathcal{Y}}_{\mathbf{q}_4} \frac{D_{(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3 + \mathbf{q}_4)/4}}{D_{\mathbf{q}_1} D_{\mathbf{q}_2} D_{\mathbf{q}_3} D_{\mathbf{q}_4}} \right\rangle - \Gamma Y_{\mathbf{q}_1 \mathbf{q}_2 \mathbf{q}_3 \mathbf{q}_4} \right\}, \quad (4c)$$

$$Y_{\mathbf{q}_1 \mathbf{q}_2 \mathbf{q}_3 \mathbf{q}_4} = \left( \frac{1}{2} - \sin^2 \delta_0 \right) (\Theta_{\mathbf{q}_1 \mathbf{q}_3}^{(2)} \Theta_{\mathbf{q}_2 \mathbf{q}_4}^{(2)} + \Theta_{\mathbf{q}_1 \mathbf{q}_4}^{(2)} \Theta_{\mathbf{q}_2 \mathbf{q}_3}^{(2)}) - \sin^2 \delta_0 (\Theta_{\mathbf{q}_1}^{(1)} \Theta_{\mathbf{q}_3}^{(1)} \Theta_{\mathbf{q}_2 \mathbf{q}_4}^{(2)} + \Theta_{\mathbf{q}_1}^{(1)} \Theta_{\mathbf{q}_4}^{(1)} \Theta_{\mathbf{q}_2 \mathbf{q}_3}^{(2)} + \Theta_{\mathbf{q}_2}^{(1)} \Theta_{\mathbf{q}_3}^{(1)} \Theta_{\mathbf{q}_1 \mathbf{q}_4}^{(2)} + \Theta_{\mathbf{q}_2}^{(1)} \Theta_{\mathbf{q}_4}^{(1)} \Theta_{\mathbf{q}_1 \mathbf{q}_3}^{(2)} - 2\Theta_{\mathbf{q}_1}^{(1)} \Theta_{\mathbf{q}_2}^{(1)} \Theta_{\mathbf{q}_3}^{(1)} \Theta_{\mathbf{q}_4}^{(1)}), \quad (4d)$$

where we introduced the angular averages  $\Theta_{\mathbf{q}_i}^{(1)} = \langle \tilde{\mathcal{Y}}_{\mathbf{q}_i} D_{\mathbf{q}_i}^{-1} \rangle$ ,  $\Theta_{\mathbf{q}_i \mathbf{q}_j}^{(2)} = \langle \tilde{\mathcal{Y}}_{\mathbf{q}_i} \tilde{\mathcal{Y}}_{\mathbf{q}_j} D_{\mathbf{q}_i}^{-1} D_{\mathbf{q}_j}^{-1} \rangle$ , and defined  $\eta_{\mathbf{q}} = \frac{1}{2} \mathbf{v}_f \cdot \mathbf{q}$ , and  $D_{\mathbf{q}} = \varepsilon_m + \Gamma + i(\mu B + \eta_{\mathbf{q}})$ . We introduced  $\tilde{\mathcal{Y}}_{\mathbf{q}} = \mathcal{Y} + \mathcal{Y}_{i, \mathbf{q}}$  with  $\mathcal{Y}_{i, \mathbf{q}} = \Gamma \Theta_{\mathbf{q}}^{(1)}$ .

The second-order N-SC transition is determined from the GL coefficient  $\alpha(T, B; \mathbf{q}) = 0$  and depends only on  $\Gamma$ , but not on  $\delta_0$ . Thus the transition line is independent of the phase shift. An instability into the modulated FFLO state becomes possible below  $T_{\mathbf{q} \neq 0}$ , where the maximal  $B_{c2}$  is found for  $q \neq 0$ . This occurs when the GL coefficient  $\kappa$  in the  $q$  expansion of  $\alpha(T, B; \mathbf{q}) \approx \alpha_0 + \kappa q^2$  becomes negative,

$$\kappa = 2\pi T \text{Re} \sum_{\varepsilon_m > 0} \frac{1}{D_0^3} \left( \langle \mathcal{Y}^2 \eta_{\mathbf{q}}^2 \rangle + \frac{\Gamma \langle \mathcal{Y} \eta_{\mathbf{q}} \rangle^2}{\varepsilon_m + i\mu B} \right). \quad (5)$$

For  $d$ -wave SC the last term vanishes, since  $\langle \mathcal{Y} \eta_{\mathbf{q}} \rangle = 0$ . In contrast, the quartic term in the GL functional explicitly de-

pends on the scattering phase shift; see Eqs. (4c) and (4d). For example, it controls the location of the first-order transition to the USC state,  $T_p$ , which competes with the FFLO instability. In unconventional superconductors  $\langle \mathcal{Y} \rangle = \mathcal{Y}_{i,0} = 0$  and the critical point  $T_p$  is determined by a sign change of the uniform ( $\mathbf{q} = 0$ ) GL coefficient  $\beta$ ,

$$\beta_0 = \pi T \text{Re} \sum_{\varepsilon_m > 0} \left( \frac{\langle \mathcal{Y}^4 \rangle}{D_0^3} - \frac{\Gamma(1 - 2 \sin^2 \delta_0)}{D_0^4} \right), \quad (6)$$

which agrees with the result derived for  $^3\text{He}$  in aerogel.<sup>17</sup> For  $\Gamma = 0$ , both  $\kappa$  and  $\beta_0$  become negative at exactly the same temperature,  $T_{\mathbf{q} \neq 0} = T_p \approx 0.5615 T_{c0}$ . Since the transition into the FFLO state has a higher critical field at any temperature  $T < T_p$ , the first-order transition is superseded by the onset of the FFLO state.<sup>10</sup>

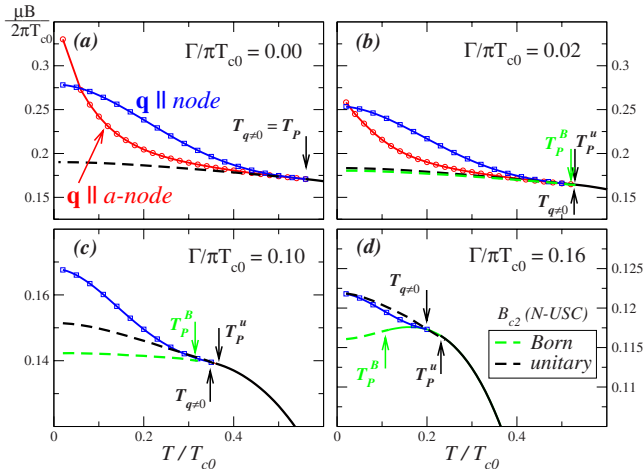


FIG. 1. (Color online) The critical fields of Pauli-limited *dirty*  $d$ -wave superconductors for transitions to USC (dashed lines, first order) and FFLO states (solid lines, second order) for  $\mathbf{q}$  along nodes (squares) and antinodes (circles). Superconductivity sets in at the highest  $B_{c2}$ . (a) Pure case: Below  $T_P = T_{q \neq 0}$  the second-order N-FFLO transition is above the N-USC transition. (b) Impurities quickly suppress  $B_{c2}(\mathbf{q} \parallel \text{antinode})$  relative to  $B_{c2}(\mathbf{q} \parallel \text{node})$ , and split  $T_P$  and  $T_{q \neq 0}$ . (c),(d) Born and unitarity impurities move  $T_P$  in opposite directions, thus  $T_P^B < T_{q \neq 0} < T_P^u$ . For Born impurities the second-order FFLO transition is above the N-USC line at all temperatures, as for the clean case. In the unitarity limit the first-order N-USC transition preempts a N-LO transition.

A comparison of  $\kappa$  and  $\beta_0$  shows that in *dirty* unconventional superconductors  $T_{q \neq 0} = T_P$  only for  $\delta_0 = \pi/4$ . For Born (B) and unitarity (u) scattering  $\beta_0$  depends on  $\delta_0$ , such that  $T_P^B$  and  $T_P^u$  shift in opposite directions relative to  $T_{q \neq 0}$ , hence  $T_P^B < T_{q \neq 0} < T_P^u$  as shown in Fig. 1. The latter inequality shows that for strong scatterers Pauli limiting leads to a first-order transition into the USC state at high fields or low temperatures in the  $B$ - $T$  phase diagram. As the system becomes dirtier (lifetime  $\tau_N$  decreases), these characteristic temperatures are suppressed to zero in the following order:  $T_P^B \rightarrow 0$  at  $\Gamma/\pi T_{c0} \geq 0.18$ ,  $T_{q \neq 0} \rightarrow 0$  at  $\Gamma/\pi T_{c0} \geq 0.20$ , and  $T_P^u \rightarrow 0$  for  $\Gamma/\pi T_{c0} \geq 0.22$ . For larger  $\Gamma$  the N-USC transition line is of second order at all  $T$ .

Figure 1 gives the upper critical field lines for different states. Second-order transitions are found by the largest spatial modulation vector  $q \equiv Q$  that maximizes  $B_{c2}$ . In clean  $d$ -wave SC (Refs. 15 and 18–20) the modulation is along a gap maximum (antinode) at low  $T/T_{c0} < 0.06$ , and along a gap node for  $0.06 < T/T_{c0} < 0.56$ ; see Fig. 1(a). However, already for small impurity scattering,  $\Gamma/\pi T_{c0} \geq 0.02$ , the critical field for  $\mathbf{q} \parallel \text{antinode}$  is lowered below  $B_{c2}^{\text{antinode}}$ , and the stable configuration is with  $\mathbf{q} \parallel \text{node}$  over the entire range of existence of the FFLO state; see Fig. 1(b).

Determining the first-order transition lines of  $B_{c2}$  requires a self-consistent calculation of the full free-energy functional, the details of which will be given elsewhere.<sup>21</sup> We find that in the Born limit the first-order transition is always below  $B_{c2}^{\text{FFLO}}$ , in agreement with Refs. 7 and 9. In contrast, in the unitarity limit  $T_{q \neq 0} < T_P^u$  and  $B_{c2}^{\text{FFLO}}$  is below the first-order transition to the USC state; see Figs. 1(b)–1(d).

For intermediate impurity scattering, the phase diagram is

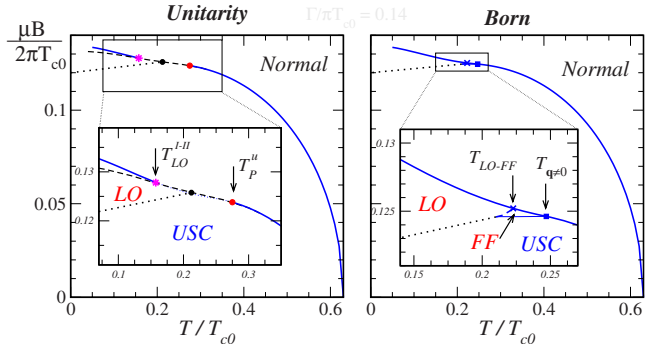


FIG. 2. (Color online) The phase diagram for  $\Gamma/\pi T_{c0} = 0.14$ . Left panel: transition into the LO state at low  $T$  becomes first order above  $T_P^{\text{I-II}}$  for unitarity impurities. There is also a region of a first-order transition into the USC state below  $T_P$ . Right panel: Born impurities result in second-order transitions. The LO state is favored in large parts of the phase diagram over the FF state, except near  $T_{q \neq 0}$  (Refs. 7 and 9). The dotted line denotes the expected transition between the LO and USC states; its exact location is beyond the GL theory.

given in Fig. 2. To determine the structure of the SC state near  $B_{c2}$ , we analyze the GL free energy, Eq. (4), for four possible phases: USC [ $\Delta(\mathbf{R}) = \Delta_{\text{USC}}$ ], FF with a single Fourier component  $\mathbf{Q}_1 = (Q, 0)$  [ $\Delta(\mathbf{R}) = \Delta_{\text{FF}} \exp(iQx)$ ], LO with  $\{\mathbf{Q}_1, \mathbf{Q}_3\} = \{(\pm Q, 0)\}$  [ $\Delta(\mathbf{R}) = \Delta_{\text{LO}2} \cos Qx$ ], and square lattice (SQ) with  $\{\mathbf{Q}_1, \mathbf{Q}_3, \mathbf{Q}_2, \mathbf{Q}_4\} = \{(\pm Q, 0), (0, \pm Q)\}$  [ $\Delta(\mathbf{R}) = \Delta_{\text{SQ}2} (\cos Qx + \cos Qy)$ ]. The  $x, y$  axes are along the gap nodes. For each phase (ph), we calculate  $\Delta \Omega_{\text{ph}}^{\text{GL}} = -\alpha^2 / \beta_{\text{ph}}$ , with  $\beta_{\text{FF}} = 2\beta_{1111}$ ,  $\beta_{\text{LO}} = \beta_{1111} + 2\beta_{1313}$ , and  $\beta_{\text{SQ}} = 0.5(\beta_{1111} + 2\beta_{1212} + 2\beta_{1313} + 2\beta_{1414} + 2\beta_{1324})$ , where  $\beta_{ijkl} = \beta(T, B; \mathbf{Q}_i, \mathbf{Q}_j; \mathbf{Q}_k, \mathbf{Q}_l)$ . Along the second-order transition line the phase with the lowest positive value of  $\beta_{\text{ph}}$  has the lowest energy.

For Born impurities (Fig. 2, right),  $\beta_{\text{ph}} > 0$  for all nonuniform states, and the LO state is favored in most of the phase

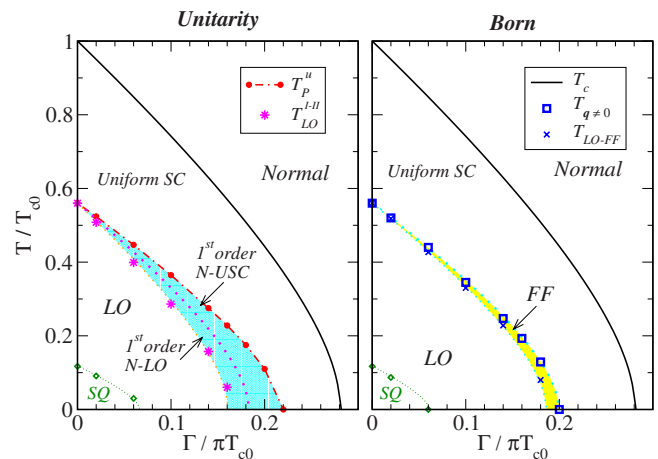


FIG. 3. (Color online) Phase transitions along the  $B_{c2}$  line in the  $\Gamma$ - $T_c$  plane. The (black) solid line is  $T_c(\Gamma)$  at  $B=0$ . For Born impurities the LO state exists for  $T < T_{q \neq 0}$ , with a small region occupied by the FF state. For unitarity scattering a first-order transition appears in the USC and LO states. At low  $T$  a square lattice FFLO state (SQ) (Ref. 19) is rapidly suppressed with increasing  $\Gamma$  (Ref. 23).

diagram except a small region below  $T_{q \neq 0}$ , where the FF phase is stabilized for the impure case.<sup>7,9</sup> In Fig. 2 along the  $B_{c2}$  line, the ratio  $\beta_{LO}/\beta_{FF}$  grows from 0.45 at  $T=0$  to 1 at  $T \approx 0.225T_{c0}$  and then to 1.5 at  $T \approx 0.25T_{c0}$ . A ratio of  $\beta_{LO}/\beta_{FF} < 1$  signals that the LO phase is stable relative to the FF phase. Additional analysis of  $\Delta\Omega^{GL}$  indicates that the FF phase is separated by a second-order transition from the USC and by first order from the LO state.

The situation is very different for strong impurities (Fig. 2, left). Following the  $B_{c2}$  line from  $T_c(B=0)$  to lower  $T$ , we reach the critical point  $T_p$ , below which the N-USC transition is of first order. At  $T \rightarrow 0$   $\beta_{LO}/\beta_{FF} = 0.26$  and the transition is second order into the LO state, but becomes first order above  $T_{LO}^{I-II}$ , where  $\beta_{LO}$  becomes negative ( $T \geq 0.15T_{c0}$ ). The details of the first-order transition, the precise location where the first-order N-USC and N-LO lines meet, and the location of the LO-USC transition line require a fully self-consistent treatment of the nonuniform problem similar to the approaches in Refs. 15 and 22.

We summarize our results in Fig. 3, where we show all states that arise along the upper critical field line for fixed  $\Gamma$ . For nonuniform states, we only consider modulations along gap nodes, since states with  $\mathbf{q} \parallel$  antinode are destabilized even faster by impurities. We find for *dirty d-wave* superconductors that the FFLO state is quite robust and survives impurity

scattering equivalent to  $\sim 40\%$  of  $T_c$  suppression or a mean-free path  $\ell$  of  $\xi_0/\ell \lesssim 0.16$ . This result is important for the generic identification of high- $B$ , low- $T$  phases in heavy-fermion<sup>24</sup> and organic superconductors.<sup>25</sup> However, a detailed analysis of those systems requires the incorporation of strong electronic correlations.

Notably, the differences between weak and strong impurity scattering are significant. In the Born limit  $T_p$  is suppressed below the onset of the nonuniform state,  $T_p^B < T_{q \neq 0}$ , and the transition is always of second order. Impurities stabilize a narrow region of the Fulde-Ferrell state just below  $T_{q \neq 0}$ . In contrast in the unitarity limit (relevant to recent experiments)  $T_{q \neq 0} < T_p^u$  and the first-order transition into the uniform state preempts a modulated state. Below  $T \sim T_{q \neq 0}$  the transition into the Larkin-Ovchinnikov state begins as a first-order line and becomes second order at lower  $T$ . Importantly, in this limit the interplay of Zeeman splitting and disorder, even without orbital effects, drives the transition between the normal and superconducting state first order.

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